QUANTUM ALGEBRAIC SYMMETRIES IN NUCLEAR AND MOLECULAR PHYSICS

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ABSTRACT

Various applications of quantum algebraic techniques in nuclear structure physics and in molecular physics are briefly reviewed.

1. Introduction

Quantum algebras (also called quantum groups) are deformed versions of the usual Lie algebras, to which they reduce when the deformation parameter q is set equal to unity. From the mathematical point of view they are Hopf algebras. Their use in physics became popular with the introduction [1,2] of the q-deformed harmonic oscillator as a tool for providing a boson realization of the quantum algebra $\mathrm{su}_q(2)$, although similar mathematical structures had already been known [3]. Initially used for solving the quantum Yang–Baxter equation, quantum algebras have subsequently found applications in several branches of physics, as, for example, in the description of spin chains, squeezed states [4], hydrogen atom and hydrogen-like spectra [5−7] rotational and vibrational nuclear and molecular spectra and in conformal field theories. By now much work has been done [8−11] on the q-deformed oscillator and its relativistic extensions [12,13], and several kinds of generalized deformed oscillators [14−16] and generalized deformed su(2) algebras [17,18] have been introduced.

Here we shall confine ourselves to applications of quantum algebras in nuclear structure physics and in molecular physics. The purpose of this short review is to provide the reader with references for further reading.

2. The $\mathrm{su}_q(2)$ rotator model

The first application of quantum algebras in nuclear physics was the use of the deformed algebra $\mathrm{su}_q(2)$ for the description of the rotational spectra of deformed [19,20] and superdeformed [21] nuclei. The Hamiltonian of the q-deformed rotator is proportional to the second order Casimir operator of the $\mathrm{su}_q(2)$ algebra. Its Taylor expansion contains powers of $J(J+1)$ (where J is the angular momentum), being similar [20] to

the expansion provided by the Variable Moment of Inertia (VMI) model. Furthermore, the deformation parameter τ (with $q = e^{i\tau}$) has been found [20] to correspond to the softness parameter of the VMI model. Through a comparison of the su$_q$(2) model to the hybrid model the deformation parameter τ has also been connected to the number of valence nucleon pairs [22] and to the nuclear deformation β [23]. Since τ is an indicator of deviation from the pure su(2) symmetry, it is not surprising that τ decreases with increasing β [23].

B(E2) transition probabilities have also been described in this framework [24]. In this case the q-deformed Clebsch–Gordan coefficients are used instead of the normal ones. (It should be noticed that the q-deformed angular momentum theory has already been much developed [24].) The model predicts an increase of the B(E2) values with angular momentum, while the rigid rotator model predicts saturation. Some experimental results supporting this prediction already exist [24]. Similarly increasing B(E2) values are predicted by a modified version [25] of the su(3) limit of the Interacting Boson Model (IBM), by the Fermion Dynamical Symmetry Model (FDSM) [26], as well as by the recent systematics of Zamfir and Casten [27].

3. Extensions of the su$_q$(2) model

The su$_q$(2) model has been successful in describing rotational nuclear spectra. For the description of vibrational and transitional nuclear spectra it has been found [28] that $J(J+1)$ has to be replaced by $J(J+c)$. The additional parameter c allows for the description of nuclear anharmonicities in a way similar to that of the Interacting Boson Model (IBM) and the Generalized Variable Moment of Inertia (GVMI) model [29]. The use of $J(J+c)$ instead of $J(J+1)$ for vibrational and transitional nuclei is also supported by recent systematics [30].

Another generalization is based on the use of the deformed algebra su$_\Phi$(2) [17,18], which is characterized by a structure function Φ. The usual su(2) and su$_q$(2) algebras are obtained for specific choices of the structure function Φ. The su$_\Phi$(2) algebra has been constructed so that its representation theory resembles as much as possible the representation theory of the usual su(2) algebra. Using this technique one can construct, for example, a rotator having the same spectrum as the one given by the Holmberg–Lipas formula [31]. A two-parameter generalization of the su$_q$(2) model, labelled as su$_{qp}$(2), has also been successfully used for the description of superdeformed nuclear bands [32].

4. Pairing correlations

It has been found [33] that correlated fermion pairs coupled to zero angular momentum in a single-j shell behave approximately as suitably defined q-deformed bosons. After performing the same boson mapping to a simple pairing Hamiltonian, one sees that the pairing energies are also correctly reproduced up to the same order. The deformation parameter used ($\tau = \ln q$) is found to be inversely proportional to the

size of the shell, thus serving as a small parameter.

The above mentioned system of correlated fermion pairs can be described *exactly* by suitably defined generalized deformed bosons [34]. Then both the commutation relations are satisfied exactly and the pairing energies are reproduced exactly. The spectrum of the appropriate generalized deformed oscillator corresponds, up to first order perturbation theory, to a harmonic oscillator with an x^4 perturbation.

If one considers, in addition to the pairs coupled to zero angular momentum, pairs coupled to non-zero angular momenta, one finds that an approximate description in terms of two suitably defined q-oscillators (one describing the $J = 0$ pairs and the other corresponding to the $J \neq 0$ pairs) occurs [35]. The additional terms introduced by the deformation have been found [35] to improve the description of the neutron pair separation energies of the Sn isotopes, with no extra parameter introduced.

q-deformed versions of the pairing theory have also been given in [36,37].

5. q-deformed versions of nuclear models

A q-deformed version of a two dimensional toy Interacting Boson Model (IBM) with $\mathrm{su}_q(3)$ overall symmetry has been developed [38,39], mainly for testing the ways in which spectra and transition probabilities are influenced by the q-deformation. The question of possible complete breaking of the symmetry through q-deformation, i.e. the transition from the $\mathrm{su}_q(2)$ limiting symmetry to the $\mathrm{so}_q(3)$ one has been examined [40,41]. It has been found that such a transition is possible for complex values of the parameter q [41]. (For problems arising when using complex q values see [42]). Complete breaking of the symmetry has also been considered in the framework of an $\mathrm{su}_q(2)$ model [43]. It has also been found [44] that q-deformation leads (for specific range of values of the deformation parameter τ, with $q = e^{i\tau}$) to a recovery of the u(3) symmetry in the framework of a simple Nilsson model including a spin-orbit term. Finally, the $\mathrm{o}_q(3)$ limit of the toy IBM model has been used for the description of ^{16}O + α cluster states in ^{20}Ne, with positive results [45].

q-deformed versions of the o(6) and u(5) limits of the full IBM have been discussed in [46–48]. The q-deformation of the su(3) limit of IBM is a formidable problem, since the $\mathrm{su}_q(3) \supset \mathrm{so}_q(3)$ decomposition has for the moment been achieved only for completely symmetric $\mathrm{su}_q(3)$ irreducible representations [49].

Furthermore a q-deformed version of the Moszkowski model has been developed [50] and RPA modes have been studied [51] in it. A q-deformed Moszkowski model with cranking has also been studied [52] in the mean-field approximation. It has been seen that the residual interaction simulated by the q-deformation is felt more strongly by states with large J_z. The possibility of using q-deformation in assimilating temperature effects is receiving attention, since it has also been found [53] that this approach can be used in describing thermal effects in the framework of a q-deformed Thouless model for supercoductivity.

In addition, q-deformed versions of the Lipkin-Meshkov-Glick (LMG) model have been developed, both for the 2-level version of the model in terms of an $\mathrm{su}_q(2)$ algebra

[54], and for the 3-level version of the model in terms of an $\mathrm{su}_q(3)$ algebra [55].

6. Anisotropic quantum harmonic oscillator with rational ratios of frequencies

The symmetries of the 3-dimensional anisotropic quantum harmonic oscillator with rational ratios of frequencies (RHO) are of high current interest in nuclear physics, since they are the basic symmetries underlying the structure of superdeformed and hyperdeformed nuclei. The 2-dimensional RHO is also of interest, in connection with "pancake" nuclei, i.e. very oblate nuclei. Cluster configurations in light nuclei can also be described in terms of RHO symmetries, which underlie the geometrical structure of the Bloch–Brink α-cluster model. The 3-dim RHO is also of interest for the interpretation of the observed shell structure in atomic clusters, especially after the realization that large deformations can occur in such systems.

The two-dimensional and three-dimensional [56] anisotropic harmonic oscillators have been the subject of several investigations, both at the classical and the quantum mechanical level (see [57,58] for references). These oscillators are examples of superintegrable systems. The special cases with frequency ratios 1:2 and 1:3 have also been considered [59]. While at the classical level it is clear that the su(N) or sp(2N,R) algebras can be used for the description of the N-dimensional anisotropic oscillator, the situation at the quantum level, even in the two-dimensional case, is not as simple. It has been proved that a generalized deformed u(2) algebra is the symmetry algebra of the two-dimensional anisotropic quantum harmonic oscillator [57], which is the oscillator describing the single-particle level spectrum of "pancake" nuclei, i.e. of triaxially deformed nuclei with $\omega_x >> \omega_y$, ω_z. Furthermore, a generalized deformed u(3) algebra turns out to be the symmetry algebra of the three-dimensional RHO [58].

7. The use of quantum algebras in molecular structure

Similar techniques can be applied in describing properties of diatomic and polytomic molecules. A brief list will be given here.

1) Rotational spectra of diatomic molecules have been described in terms of the su$_q$(2) model [60]. As in the case of nuclei, q is a phase factor ($q = e^{i\tau}$). In molecules τ is of the order of 0.01. The use of the su$_q$(2) symmetry leads to a partial summation of the Dunham expansion describing the rotational–vibrational spectra of diatomic molecules [60]. Molecular backbending (bandcrossing) has also been described in this framework [61]. Rotational spectra of symmetric top molecules have also been considered [62,63] in the framework of the su$_q$(2) symmetry.

2) Vibrational spectra of diatomic molecules have been described in terms of q-deformed anharmonic oscillators having the su$_q$(1,1) [64] or the u$_q$(2) $\supset$ o$_q$(2) [65] symmetry, as well as in terms of generalized deformed oscillators similar to the ones described in sec. 3 [66,67]. These results, combined with 1), lead to the full summation of the Dunham expansion [64,65]. A two-parameter deformed anharmonic oscillator with u$_{qp}$(2) $\supset$ o$_{qp}$(2) symmetry has also been considered [68].

3) The physical content of the anharmonic oscillators mentioned in 2) has been clarified by constructing WKB equivalent potentials (WKB-EPs) and classical equiv-

alent potentials [69] providing approximately the same spectrum. The results have been corroborated by the study of the relation between su$_q$(1,1) and the anharmonic oscillator with x^4 anharminicities [70]. Furthermore the WKB-EP corresponding to the su$_q$(1,1) anharmonic oscillator has been connected to a class of Quasi-Exactly Soluble Potentials (QESPs) [71].

4) Generalized deformed oscillators giving the same spectrum as the Morse potential [72] and the modified Pöschl–Teller potential [73], as well as a deformed oscillator containing them as special cases [74] have also been constructed. In addition, q-deformed versions of the Morse potential have been given, either by using the so$_q$(2,1) symmetry [75] or by solving a q-deformed Schrödinger equation for the usual Morse potential [76].

5) A q-deformed version of the vibron model for diatomic molecules has been constructed [77], in a way similar to that described in sec. 5.

6) For vibrational spectra of polyatomic molecules a model of n coupled generalized deformed oscillators has been built [78], containg the approach of Iachello and Oss [79] as a special case.

7) Quasi-molecular resonances in the systems ^{12}C+^{12}C and ^{12}C+^{16}O have been described in terms of a q-deformed oscillator plus a rigid rotator [80].

A review of several of the above topics, accompanied by a detailed and self-contained introduction to quantum algebras, has been given by Raychev [81].

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